PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 579–584

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Calculating transient wall heat flux from measurements of surface temperature

L. Reichelt *, U. Meingast, U. Renz

Lehrstuhl für Wärmeübertragung und Klimatechnik, Rheinisch–Westfälische Technische Hochschule Aachen, 52056 Aachen, Germany

Received 20 February 2001; received in revised form 25 April 2001

Abstract

Wall heat fluxes can be derived from time resolved measurements of the surface temperature. This paper describes an analytical approach to calculate the heat flux from an analytical solution of the one-dimensional transient energy equation with transient boundary conditions using the Laplace transformation. The results are compared to simple test cases for which the heat fluxes are given in literature. The method is used to calculate the heat flux from a fuel spray to a wall at diesel engine conditions. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Surface temperature; Wall heat flux; Laplace transformation; Analytical solution; Inverse problem

1. Introduction

In modern direct injection engines the fuel spray interacts with the piston and the cylinder walls. These interactions may affect the atomization of the spray. In an experimental set-up a high temperature high pressure injection chamber is used to examine spray wall interactions. Since the heat flux to the wall cannot be measured directly, indirect measurement techniques have to be employed. Fast surface thermocouples are used to measure the surface temperature at three positions within the impingement region of the spray. Fig. 1 shows a sketch of the experimental setup and the thermocouples. The thermocouple is soldered at its top to the wall and surrounded by an air gap. Because of this insulation the heat conduction in the thermocouple can be assumed to be one-dimensional. Details of the experimental setup can be found in [1]. To calculate the heat flux from these measurements an analytical approach is proposed. The Laplace transformation is used and the heat flux is obtained from the temperature gradient at the surface. Thus it is not necessary to numerically solve the energy equation on the whole domain and computational efforts and errors due to numerical inaccuracy are reduced.

2. Governing equations

Since the problem is considered to be one-dimensional the transient energy equation is

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\xi^2},\qquad \tau=\frac{ta}{\delta^2},\quad \xi=\frac{x}{\delta},\quad \theta=\vartheta-\vartheta_\delta. \tag{1}$$

Assuming an uniform temperature distribution prior the impingement of the spray and a constant temperature ϑ_δ at $x=\delta$ the initial and boundary conditions are:

$$\theta(\xi,0)=0,\qquad \theta(0,t)=\vartheta_S(t)-\vartheta_\delta=\theta_S(t),\qquad \theta(1,t)=0. \tag{2}$$

* Corresponding author. Tel.: +49-241-80-5537; fax: +49-241-8888-143.

E-mail address: reichelt@wuek.rwth-aachen.de (L. Reichelt).

PII: S0017-9310(01)00157-0

Nomenclature

a	thermal diffusivity
c	heat capacity
F	frequency
$\mathscr{L}$	Laplace transformation operator
p	substitution: $p^2 = s$
$\dot{q}''$	wall heat flux
s	parameter in frequency-domain
t	time
x	coordinate
Greek letters	
δ	thickness of the wall
$\Delta\tau$	dimensionless time-step
$\Delta\vartheta$	temperature difference
ϑ	temperature
θ	temperature difference: $\theta = \vartheta - \vartheta_\delta$
$\overline{\theta}$	transformed temperature
λ	thermal conductivity
ϱ	density
τ	dimensionless time
ξ	dimensionless coordinate
Indices	
M	averaged
S	surface
δ	position $x = \delta$

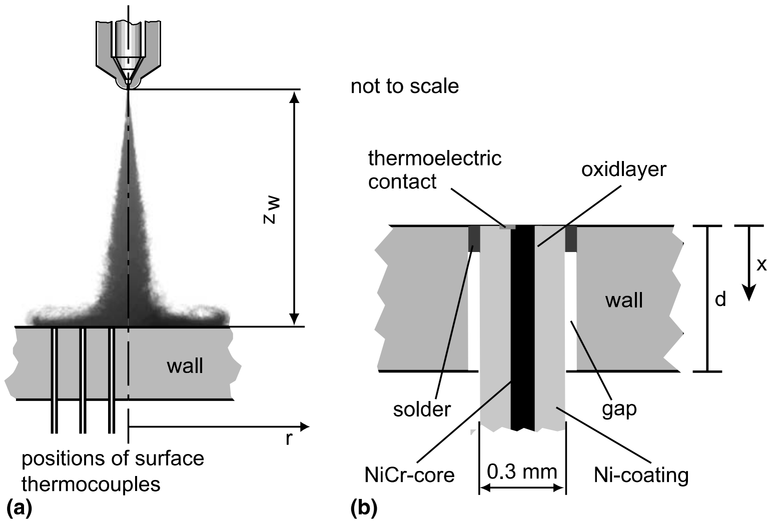

Fig. 1. Sketch of the experimental setup and a surface thermocouple.

If the transient temperature distribution at the surface is known, the instantaneous heat flux can be calculated:

$$\dot{q}'' = -\lambda \frac{\partial \vartheta}{\partial x}(0, t) = -\frac{\lambda}{\delta} \cdot \frac{\partial \theta}{\partial \xi}(0, \tau). \tag{3}$$

A positive sign to the heat flux denotes heat flux into the wall. Using the Laplace transformation [2] this partial differential equation can be transformed into an ordinary differential equation:

$$s\overline{\theta}(\xi, s) - \theta(\xi, 0) = s\overline{\theta}(\xi, s) = \frac{\partial^2 \overline{\theta}}{\partial \xi^2}. \tag{4}$$

Using $p^2 = s$ the general solution for this equation is

$$\overline{\theta}(\xi, s) = A\mathrm{e}^{p\cdot\xi} + B\mathrm{e}^{-p\cdot\xi}. \tag{5}$$

Using the transformed boundary conditions the solution becomes

$$\overline{\theta}(\xi, s) = \overline{\theta}_\mathrm{S}(s) \cdot \frac{\mathrm{e}^{-p\xi} - \mathrm{e}^{p(\xi-2)}}{1 - \mathrm{e}^{-2p}}. \tag{6}$$

In order to calculate the heat flux the gradient at the surface is needed according to Eq. (3):

$$\frac{\partial \overline{\theta}}{\partial \xi}(0, s) = -\overline{\theta}_\mathrm{S}(s) p \cdot \frac{1 + \mathrm{e}^{-2p}}{1 - \mathrm{e}^{-2p}} = -s\overline{\theta}_\mathrm{S}(s) \cdot \frac{1 + \mathrm{e}^{-2p}}{p(1 - \mathrm{e}^{-2p})}. \tag{7}$$

Expanding the denominator according to

$$\frac{1}{1-x} = 1 + \sum_{n=1}^{\infty} x^n, \quad x < 1 \tag{8}$$

the surface gradient may be expressed as a series:

$$\frac{\partial\overline{\theta}}{\partial\xi}(0,s) = -s\overline{\theta}_S(s)\cdot\frac{1+e^{-2p}}{p}\left(1+\sum_{n=1}^{\infty}e^{-2np}\right)$$
$$= -s\overline{\theta}_S(s)\cdot\left(\frac{1}{p}+\frac{e^{-2p}}{p} + \sum_{n=1}^{\infty}\left(\frac{e^{-2np}}{p}+\frac{e^{-(2n+2)p}}{p}\right)\right)$$
$$= \underbrace{-s\overline{\theta}_S(s)}_{\overline{F}_1(s)}\cdot\underbrace{\left(\frac{1}{p}+2\sum_{n=1}^{\infty}\frac{e^{-2np}}{p}\right)}_{\overline{F}_2(s)}. \quad (9)$$

Using transformation tables given for example in [2,3], both terms of this expression may be transformed back:

$$F_1(t) = -\frac{d\theta_S}{dt}(t),$$
$$F_2(t) = \frac{1}{\sqrt{\pi t}}\left(1+2\sum_{n=1}^{\infty}e^{-n^2/t}\right). \quad (10)$$

Using the convolution-theorem [2]

$$\mathscr{L}^{-1}(\overline{F}_1(s)\cdot\overline{F}_2(s)) = \int_0^t F_1(t^*)\cdot F_2(t-t^*)\,dt^* \quad (11)$$

the equation for the surface gradient can be derived:

$$\frac{\partial\theta}{\partial\xi}(0,\tau) = -\int_0^{\tau}\frac{d\theta_S}{d\tau}(\tau^*)\cdot\frac{1}{\sqrt{\pi(\tau-\tau^*)}} \times\left(1+2\sum_{n=1}^{\infty}e^{-(n^2/(\tau-\tau^*))}\right)d\tau^*. \quad (12)$$

Applying Eq. (3) the wall heat flux can be determined:

$$\dot{q}''(\tau) = \frac{\lambda}{\sqrt{\pi}\cdot\delta}\int_0^{\tau}\frac{d\vartheta_S}{d\tau^*}(\tau^*)\cdot\frac{1}{\sqrt{\tau-\tau^*}} \times\left(1+2\sum_{n=1}^{\infty}e^{-(n^2/(\tau-\tau^*))}\right)d\tau^*. \quad (13)$$

3. Implementation

The derived expression for the heat flux (Eq. (13)) is evaluated numerically using the measured surface temperatures ϑ_S. These values are sampled at discrete time intervals $\tau_i = i\cdot\Delta\tau$.

Then the integral in Eq. (13) can be split into

$$\begin{aligned}\dot{q}''(\tau_i) = {}& \frac{\lambda}{\sqrt{\pi}\cdot\delta}\int_0^{\tau_1}\frac{d\vartheta_S}{d\tau^*}(\tau^*) \times\frac{1}{\sqrt{\tau_i-\tau^*}}\left(1+2\sum_{n=1}^{\infty}e^{-(n^2/(\tau_i-\tau^*))}\right)d\tau^*\\ &+\frac{\lambda}{\sqrt{\pi}\cdot\delta}\int_{\tau_1}^{\tau_2}\frac{d\vartheta_S}{d\tau^*}(\tau^*)\cdot \times\frac{1}{\sqrt{\tau_i-\tau^*}}\left(1+2\sum_{n=1}^{\infty}e^{-(n^2/(\tau_i-\tau^*))}\right)d\tau^*\\ &+\cdots+\frac{\lambda}{\sqrt{\pi}\cdot\delta}\int_{\tau_{i-1}}^{\tau_i}\frac{d\vartheta_S}{d\tau^*}(\tau^*)\frac{1}{\sqrt{\tau_i-\tau^*}} \times\left(1+2\sum_{n=1}^{\infty}e^{-(n^2/(\tau_i-\tau^*))}\right)d\tau^*.\end{aligned} \quad (14)$$

For small values $\tau \ll 1$ the sum may be neglected, hence

$$\dot{q}''(\tau_i) = \frac{\lambda}{\sqrt{\pi}\delta}\sum_{k=0}^{i-1}\int_{\tau_k}^{\tau_{k+1}}\frac{d\vartheta_S}{d\tau^*}(\tau^*)\cdot\frac{1}{\sqrt{\tau_i-\tau^*}}d\tau^*. \quad (15)$$

To get a good approximation of the derivative in the interval $[\tau_k, \tau_{k+1}]$ the Taylor series expansion is used:

$$\frac{d\vartheta_S}{d\tau^*}(\tau^*) = \frac{d\vartheta_S}{d\tau^*}(\tau_M) + (\tau^*-\tau_M)\frac{d^2\vartheta_S}{d\tau^{*2}}(\tau_M)$$
$$\tau_M = \frac{\tau_k+\tau_{k+1}}{2} = \Delta\tau\frac{2k+1}{2}. \quad (16)$$

Using

$$\vartheta'_{S,k} = \frac{d\vartheta_S}{d\tau^*}(\tau_M) = \frac{\vartheta_{S,k+1}-\vartheta_{S,k}}{\Delta\tau}$$
$$\vartheta''_{S,k} = \frac{d^2\vartheta_S}{d\tau^{*2}}(\tau_M) = \frac{(\vartheta_{S,k+2}-\vartheta_{S,k+1})-(\vartheta_{S,k}-\vartheta_{S,k-1})}{2(\Delta\tau)^2}. \quad (17)$$

Eq. (15) may be written as

$$\dot{q}''(\tau_i) = \frac{\lambda}{\sqrt{\pi}\delta}\sum_{k=0}^{i-1}\int_{\tau_k}^{\tau_{k+1}}\left(\vartheta'_{S,k}+(\tau^*-\tau_M)\vartheta''_{S,k}\right)\cdot\frac{1}{\sqrt{\tau_i-\tau^*}}d\tau^*. \quad (18)$$

Integrating this equation yields the final equation to calculate the heat flux from the measured data:

$$\dot{q}''(\tau_i) = 2\frac{\lambda}{\delta}\sqrt{\frac{\Delta\tau}{\pi}}\sum_{k=0}^{i-1}\left(\left(\vartheta'_{S,k}+\vartheta''_{S,k}\Delta\tau\left(i-\frac{2k+1}{2}\right)\right) \times R_{i,k} - \vartheta''_{S,k}\frac{\Delta\tau}{3}S_{i,k}\right),$$
$$R_{i,k} = (i-k)^{1/2}-(i-k-1)^{1/2}, \quad (19)$$
$$S_{i,k} = (i-k)^{3/2}-(i-k-1)^{3/2}.$$

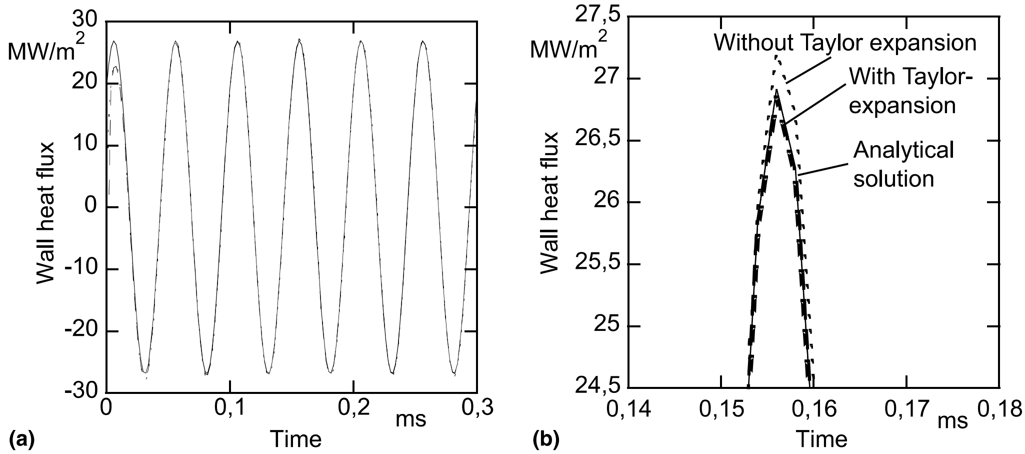

Fig. 2. Surface heat fluxes of a seminfinite wall derived with different solutions ($a = 3.9 \times 10^{-6}$ m^2 s^{-1}, $\lambda = 15$ W m^{-1} K^{-1}, $f = 20$ kHz).

4. Comparison with analytical results

For a semi-infinite body whose surface temperature is changing according to

$$\vartheta_S(t) = \vartheta_{S,0} + \Delta\vartheta \sin(2\pi t f), \quad (20)$$

where f denotes the frequency of the temperature changes and $\vartheta_{S,0}$ and $\Delta\vartheta$ the average surface temperature and the amplitude of the temperature changes, the surface heat flux may be exactly calculated as [4]

$$\dot{q}''(t) = \lambda \Delta\vartheta \sqrt{\frac{\pi f}{a}} (\sin(2\pi t f) + \cos(2\pi t f)). \quad (21)$$

Fig. 2 shows a comparison between heat fluxes calculated from the solution derived above and the exact solution from Eq. (21). The parameters are $f = 20$ kHz, $a = 3.9 \times 10^{-6}$ m^2 s^{-1} and $\lambda = 15$ W m^{-1} K^{-1}. For the numerical calculations the data has been sampled at intervals of $\Delta t = 2 \times 10^{-6}$ s. It can be seen that after a short settling time the differences between the different solutions are very small and that using the Taylor expansion the errors may be significantly reduced at high frequencies. The same calculation with a frequency of $f = 500$ Hz shows no difference whether the Taylor expansion is used or not.

5. Results

The method presented here has been used to determine the surface heat flux during spray/wall interaction under diesel engine conditions. The data were sampled at a data rate of 5 MHz using a digital oscilloscope and processed in a PC using the DAQ Software LabView 5.0 [5]. To suppress the noise from the Analog–Digital–Converter, a digital lowpass filter is used. A second-order digital Butterworth filter [5,6] with a cut-off frequency of 20 kHz provided good results. Fig. 3 shows the amplitude response of the filter. A n-dodecane spray is impinging orthogonally on a wall. The ambient gas pressure and the fuel injection pressure are set to $p_{\mathrm{Gas}} = 38.5$ bar and $p_{\mathrm{inj}} = 800$ bar, respectively. The temperatures of the gas and the wall are set to $\vartheta_{\mathrm{Gas}} = \vartheta_{\mathrm{W}} = 600$ K, the distance between the nozzle and the wall is $z_{\mathrm{W}} = 30$ mm. Fig. 4 shows the original temperature data at three different radial positions (see Fig. 1) and the lowpass filtered data. Obviously the noise is effectively suppressed. Since only the temporal derivative of the temperature is needed, it is sufficient to measure the change of the surface temperature relative to the temperature before the start of the injection. Thus all the traces start at a relative temperature of 0 K.

Fig. 5 shows the heat fluxes calculated from the data shown in Fig. 4. The parameters used are: $\Delta t = 2 \times 10^{-7}$ s, $a = 6.5 \times 10^{-6}$ m^2 s^{-1}, $\lambda = 28.1$ W m^{-1} K^{-1} and $\delta = 10$ mm. To get averaged heat flux values which can be used to validate numerical simulations of the spray–wall interactions [1] the heat fluxes have also been

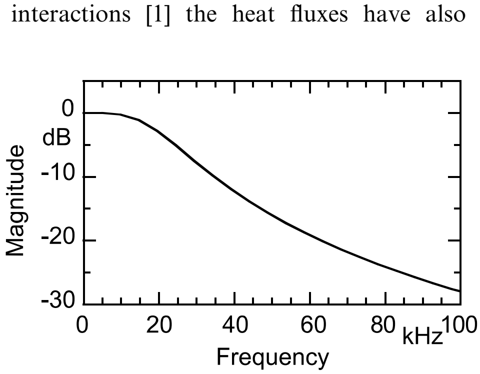

Fig. 3. Amplitude response of the butterworth filter.

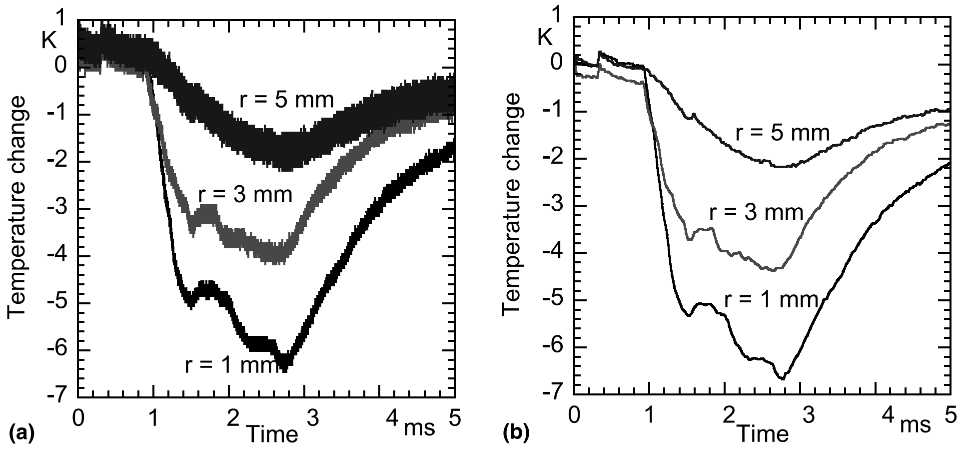

Fig. 4. Unfiltered (left) and filtered (right) temperature data.

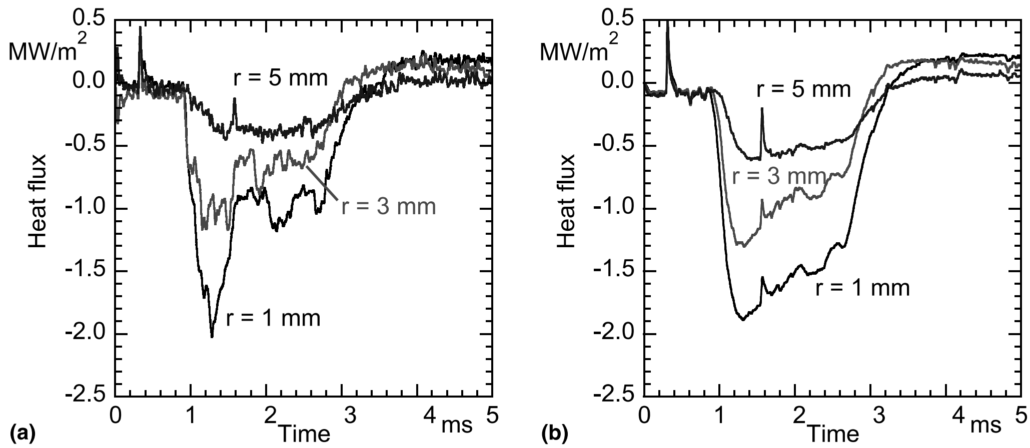

Fig. 5. Wall heat fluxes: left: single measurement, right: heat fluxes from averaged temperature data ($p_{Gas} = 38.5$ bar, $p_{inj} = 800$ bar, $\vartheta_{Gas} = \vartheta_{Wall} = 600$ K, $z_W = 30$ mm).

calculated from averaged temperature measurements derived from 50 injections. The two peaks at times 0.3 and 1.6 ms result from electromagnetic noise from the injector at the beginning and the end of the injection and can not be further suppressed by filtering without altering the data.

6. Conclusions

An analytical approach to calculate heat fluxes from time resolved measurements of the surface temperature has been introduced. The method proposed here shows good agreement with analytical results and needs only little computational efforts. By the use of this method wall heat fluxes of fuel sprays at diesel engine conditions have been calculated. The understanding of the process during wall contact is enhanced and the results support the interpretation of data gained with optical flow measurements.

Acknowledgements

Part of this work has been granted by the German Ministry of Education and Research (BMBF) and Ford Forschungszentrum Aachen GmbH under grant no. 13N7182/1.

References

[1] U. Meingast, M. Staudt, L. Reichelt, U. Renz, F.-A. Sommerhoff, Analysis of spray/wall interaction under diesel

engine conditions, SAE Technical Paper 2000-01-0272 (2000).
[2] I.N. Bronstein, K.A. Semendjajew, G. Musiol, H. Mühlig, Taschenbuch der Mathematik, Verlag Harry Deutsch, Frankfurt am Main, Thun, 1999.
[3] H.S. Carslaw, J.C. Jaeger, Conduction of Heat in Solids, second ed., Oxford University Press, Oxford, 1959.
[4] E. Eckert, R.M. Drake, Analysis of Heat and Mass Transfer, McGraw-Hill, New York, 1972.
[5] LabView 5.0 Users Guide, National Instruments, Austin, TX, USA, 1996.
[6] C.B. Rorabaugh, Digital Filter Designer's Handbook, McGraw-Hill, New York, 1993.